Dependency/Competition in Corn Yield: A Proxy for Land Use Change

October 2009

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Setup

We investigate the effect of biofuels on land use change via U.S. corn production data.

- Agricultural models are used to estimate the effect of biofuel production on crop production $\Rightarrow$ land use change
- Statistical determination of the influence of biofuel production is non-standard $\Rightarrow$ total crop use is always the sum of the different uses
- These (induced) distributions are *Compositional Distributions* $\Rightarrow$ Methodology for Dependency/Competition among Compositional Distributions

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Outline

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Illustrations

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Energy Independence and Security Act

- EISA-2007 mandates an increase in ethanol production to 36 billion gallons per year by 2022.
- Ethanol production has increased more than 5000 percent since 1980
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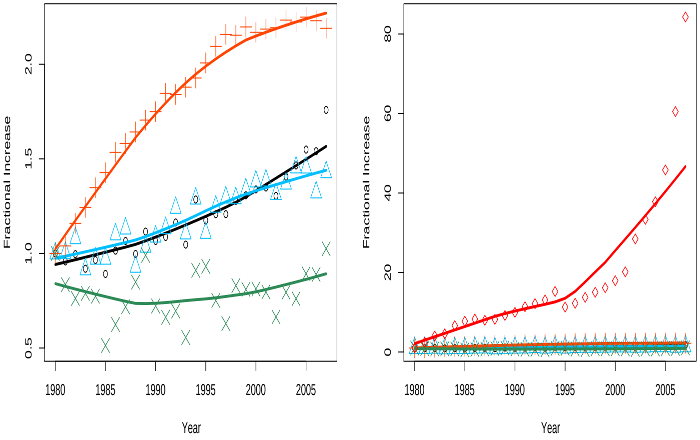
Fractional Increase



Environmental Impacts

- Net energy budget
- Competition with corn-based commodities
- Greenhouse emissions
- $\Rightarrow$ Competition within distribution of Corn Yield constitutents

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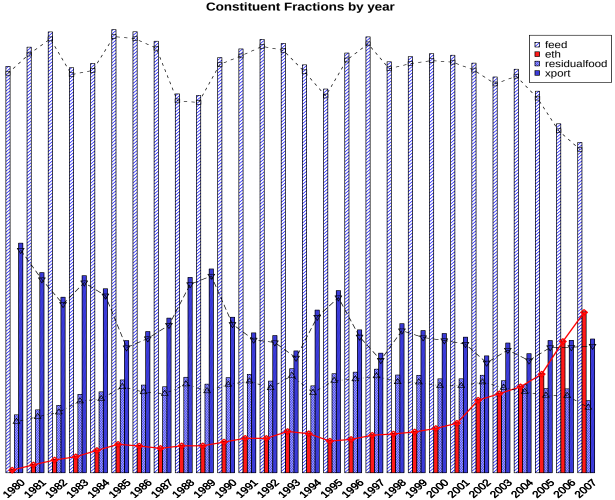
Constituents of Corn Yield



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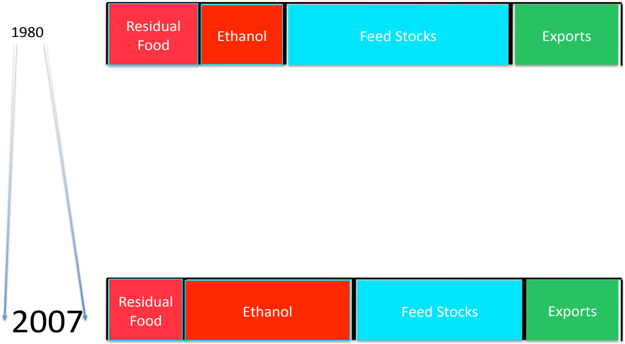

Aitchison Notation

Let

$$\mathbf{x} = (x_1, ..., x_k) \tag{1}$$

be a basis or open vector of positive quantities
In this example

$$\mathbf{x} = (x_{eth}, x_{rfood}, x_{feed}, x_{xport})$$

(in bushels) corn of: ethanol production, residual food stock, feed stock, and exports.

Aitchison Notation

Let

$$y_j = x_j / \sum_j^k x_j \tag{2}$$

$\mathbf{y} = (y_1, ..., y_k)$ the vector of fractions.
Aitchison defines $y_{k+1} = 1 - \sum_j^k y_j$;
Here $\sum_j^k y_j = 1$

Aitchison Notation

A (log-ratio) transformation sets

$$v_{\substack{j\\m}} = log(\frac{y_j}{y_m}) = log\ y_j - log\ y_m \tag{3}$$

in a slight modification of Aitchison's notation (where $v_j = log(y_j/y_{k+1})$).

Modifying Aitchison Notation

- The total is fixed and known $\Rightarrow$ the residual is y_{k+1}=0 and Aitchison's v_j is undefined
- In the original notation v_j is the log of the relative fraction of constituent j to the residual component of the basis
- $v_{\cdot m}$, maps $\mathbb{S}^{m,k} = \{\mathbf{y}_{-m}, y_m\}$ to $\mathbb{R}^{k-|m|}$

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Why Transform?

(Natural) Dirichlet model for $(y_1, ..., y_k)$; $\sum_j y_j = 1$; $y_j > 0 \; \forall j$ is:

$$dF(\mathbf{y}) \propto (1 - \sum_j y_j)^{\alpha_{k+1}-1} \cdot \prod_j y_j^{\alpha_j - 1} \tag{4}$$

with parameters $\alpha = (\alpha_1, ..., \alpha_{k+1})$
Insufficient for non-*neutral* proportions

Why Transform?

(Generalization) Liouville distribution is:

$$dF \propto h(\sum_j y_j) \prod y_j^{\alpha_j - 1} \tag{5}$$

with $\alpha_j > 0$ (as before) and g some function.
Note that when $h(t) = 1 - t$ the Liouville distribution is the special case Dirichlet distribution with $\alpha_{k+1} = 1$
Thus h is an additional parameter of interest for estimation

Aitchison Approach

- Aitchison fits a log-normal distribution on $\mathbf{v}$
- Σ is sufficient for dependency in the log-normal distribution

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Aitchison Approach

- Under a composition

$$\Sigma_{\mathbf{v}} \propto diag(\omega_1, ..., \omega_k) + \omega_{k+1}, \tag{6}$$

- $\Sigma_{\mathbf{v}}$ is constrained to the positive orthant and proportional to the units of the residual component

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Aitchison Approach

- Aitchison uses a likelihood ratio test
- Test statistic is iteratively estimated due to the constraints on the support of the parameter space
- $\Sigma_{\mathbf{v}} \geq \mathbf{0}$ and proportional to the choice of y_{k+1}

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Our Approach

Multivariate Version of KS distance:

$$D_{n,k} = \sup_{\mathbf{t}} |F_n(\mathbf{t}) - F(\mathbf{t})| \tag{7}$$

For $\mathbf{t} = (t_1, t_2, ...)$ the distance is a probability measure on Kendall's distributions...chi-square convergence does not hold (Nelsen 2003).

Our Approach

A Kolmogorov-Smirnov statistic (distance) for multivariate independence can be written:

$$D^{\Pi}_{n,k} = \sup_{\mathbf{t}} |F_n(\mathbf{t}) - \prod_j F_j(t_j)|. \tag{8}$$

Under independence the distance converges to zero (via Glivenko-Cantelli), but distributionally for $k \geq 2$?

Our Approach

- Let $\mathbf{u} = (u_1, ..., u_k)$, where each $u_j = F_j(v_j)$
- Let the joint distribution for $\mathbf{v}$ be $F(\mathbf{v})$
- The *copula* for $\mathbf{u}$ is

$$C(\mathbf{u}) = F(F_1(v_1), ..., F_k(v_k)) \tag{9}$$

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With $C_n(\cdot)$ a multivariate version of the *empirical copula*:

$$C_n(\mathbf{u}) = \frac{\#\{\mathbf{t} \mid t_1 \leq F_1^{-1}(u_1), ..., t_k \leq F_k^{-1}(u_k)\}}{n} \tag{10}$$

Our Approach

- Fit a *Dirichlet* distribution (i.e. estimate $\hat{\alpha} = (\hat{\alpha}_1, ..., \hat{\alpha_k})$ for $\alpha = (\alpha_1, ..., \alpha_k)$) to the composition data $\mathbf{y}$.
- Generate T *Dirichlet* replicates, parameter $\hat{\alpha}$, each of dimension $n \times k$: $(\mathbf{y}^{\hat{\alpha},1}, ..., \mathbf{y}^{\alpha,T})$.
- Compute $m = 1...k$ versions of Aitchison's log-ratios on the replicates: $\mathbf{v}_m^{\hat{\alpha},1} ..., \mathbf{v}_m^{\hat{\alpha},T}$
- For $m = 1..k$ compute $D_{n,k}^{\Pi,1}{}_m, ..., D_{n,k}^{\Pi,T}{}_m$ of

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- Yields a distribution for the statistic under an independence hypothesis among the compositions...
- m versions of $D^{\Pi,1}_{\frac{n,k}{m}}, \ldots, D^{\Pi,T}_{\frac{n,k}{m}}$ are proxies for tests of *complete subcompositional independence...*
- Calculating on the log-ratios ($\mathbf{v}$) of the replicates, and not the *Dirichlet* draws picks each of m components to serve as 'residual' (via the basis $\mathbf{x}$ or composition $\mathbf{y}$) without requiring m estimates of α and m-fold random draws.

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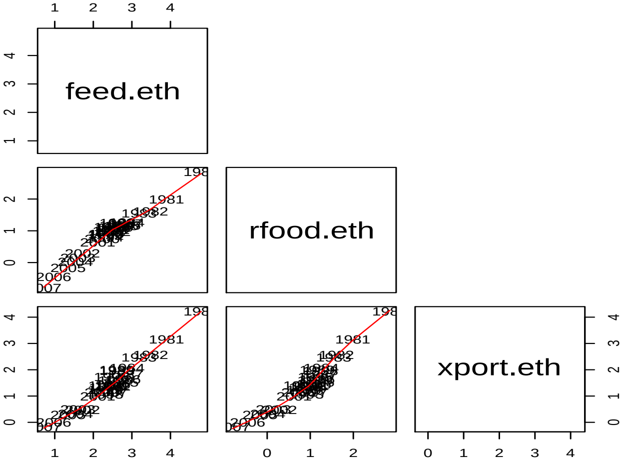
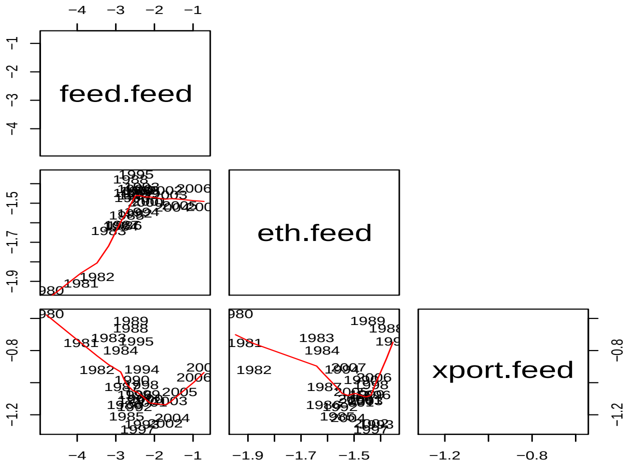
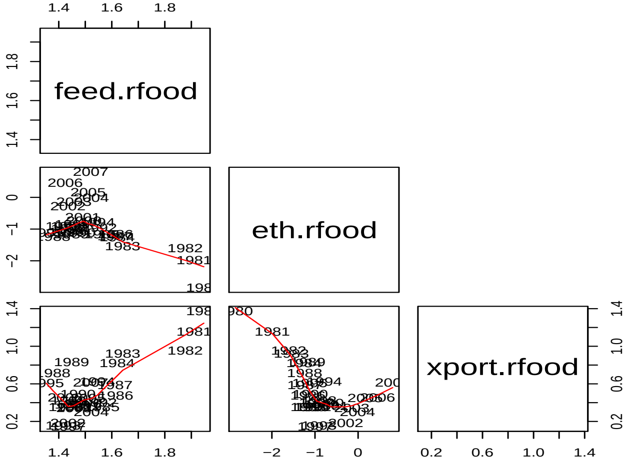
W.R.T. Ethanol



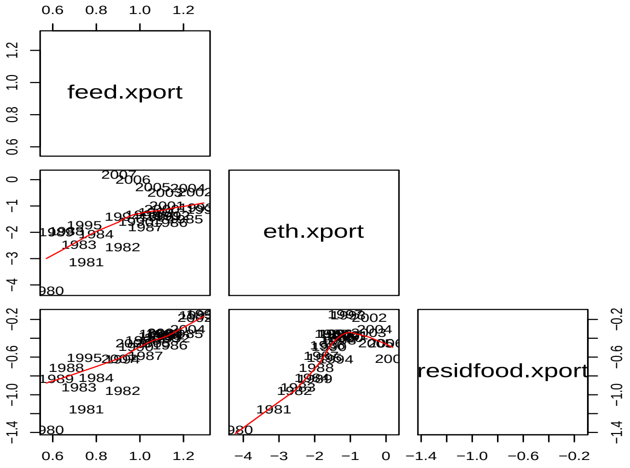
W.R.T. Feed



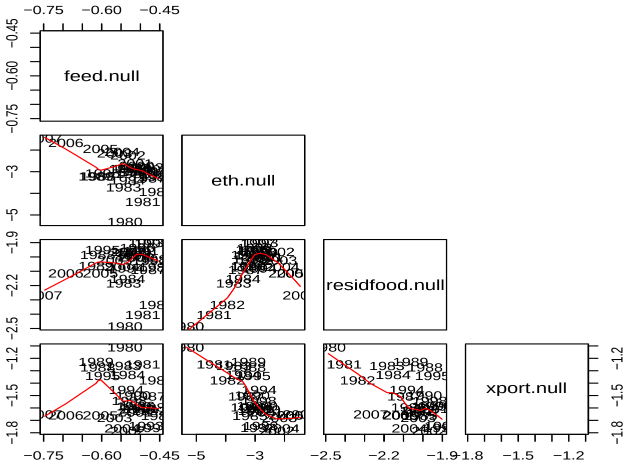
W.R.T. Food



W.R.T. Export


feed.xport
eth.xport
residfood.xport
0.6
0.8
1.0
1.2
−4
−3
−2
−1
0
−1.4
−1.0
−0.6
−0.2

W.R.T. Null


feed.null
eth.null
residfood.null
xport.null
−0.75
−0.60
−0.45
−5
−3
−1.9
−2.2
−2.5
−1.2
−1.5
−1.8

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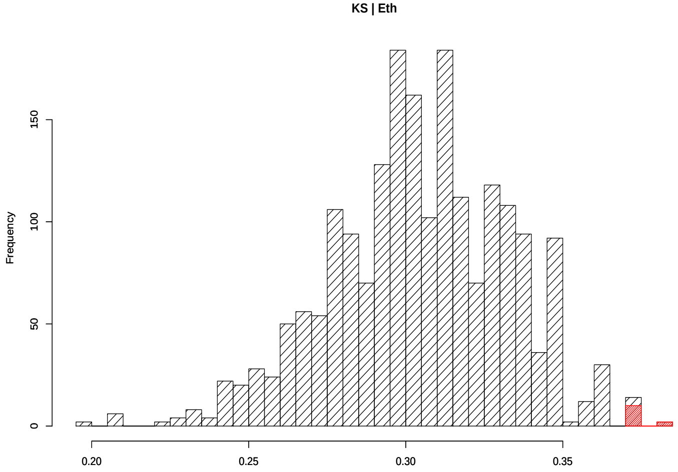

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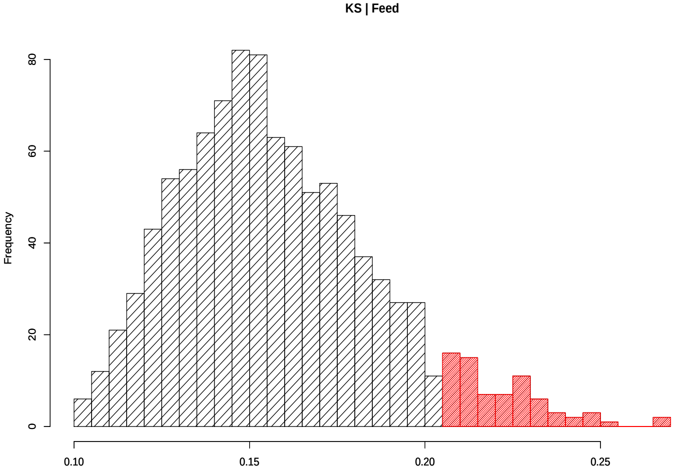
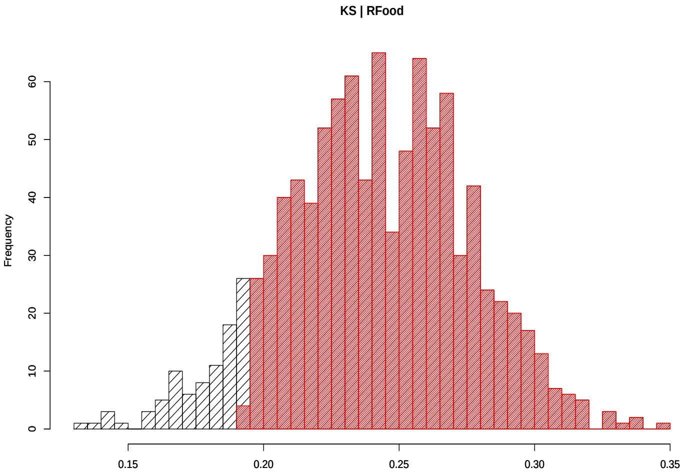
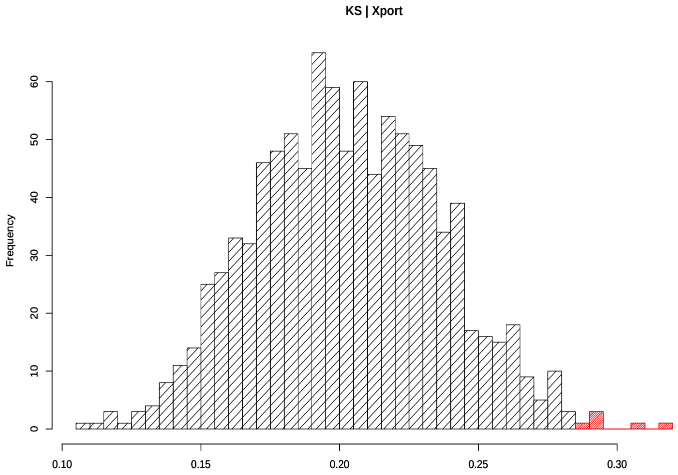


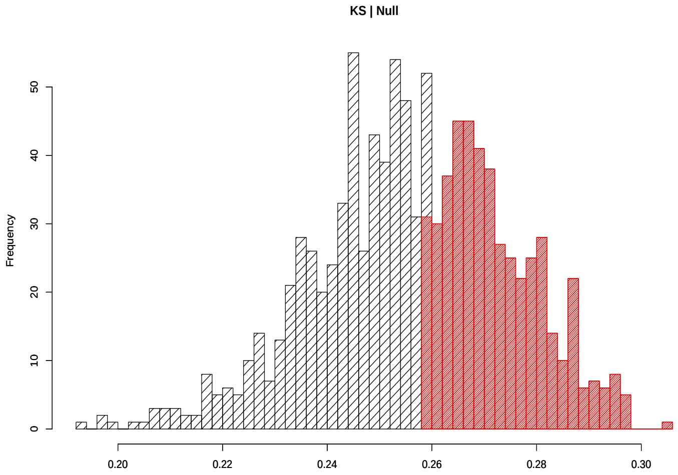
W.R.T. Residual Food



W.R.T. Export



W.R.T. Null



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- Empirical Prob Integral Transform : : Order statistics $\Rightarrow$ Invariant to increasing (log-ratio) transform.
- In high dimensions Aitchison's method 'tail-migrates'
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