Bayesian Multivariate Extreme Value Thresholding for Environmental Hazards

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Outline



Introduction and Motivation

- A Motivating Example
- Thresholding Data
- 2 Data
 - Events
 - Vulnerabilities
- 3 Methodology
 - Approaches to Thresholding

4 Results

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A Motivating Example Thresholding Data

High Risk Hotspot

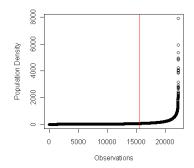
Between 1994-1998: Volcano eruption in Rabaul, Cyclone Justin in the Milne Bay (SE from map selection), and El Niño-induced drought



A Motivating Example Thresholding Data

One Variable

In the univariate setting thresholding is straightforward...



 ..the separation of data into regular-valued and extreme-valued portions.

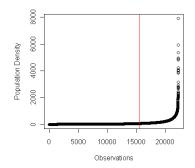
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A Motivating Example Thresholding Data

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A Motivating Example Thresholding Data

Multivariate Data

Taking multivariate \mathbf{q} , say, we want to return the set \mathcal{T} such that

$$\mathcal{T} = \{t | F(\mathbf{T} > \mathbf{t}) > \mathbf{c}\}$$
(1)

Censor the data:

$$\mathcal{T} \supset \mathcal{T}_* = \{ \mathbf{t} \mid t_i > \mathbf{c}, \forall i \}$$
(2)

And the output is: *F* for *i* = 1, 2 is $F(T \le t_*) = F_1 + F_2 - F_1F_2$ and $F_1 = Pr(T \le t_*)$; $F_2 = F_1 = Pr(T \le t | T > t_*)$

A Motivating Example Thresholding Data

Multivariate Data

In the Multivariate setting this is to fit some contour that partitions multivariate data into

- Regular valued
- Extreme valued

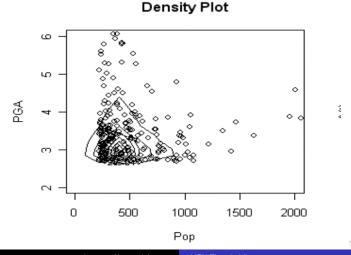
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Introduction and Motivation

Data Methodology Results A Motivating Example Thresholding Data

Pop vs. PGA



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Events Vulnerabilities

Global Natural Disaster Risk Hotspots

Worldwide data has been gridded to $1\frac{1}{2}^{\circ}$ boxes for 8 predictor variables.

• GDP

- Population
- Peak Ground Acceleration (PGA)
- Floods
- Cyclones
- Drought
- Volcanoes
- Landslides

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Introduction and Motivation Data Events Methodology Vulnera Results

2003 Global Natural Disaster Risk Hotspots Data

Incidence Maps, gridded to 1.5° lat-lon, 8 variables

- Floods
- Volcano
- Drought
- Earthquake
- GNP: 1990 Gross National Product in US dollars
- Population: Gridded population count (estimate) 1995

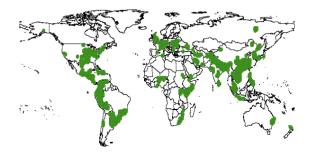
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Events Vulnerabilities

Floods

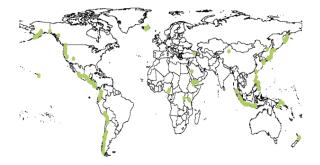
.9 ptile of Flood counts



Events Vulnerabilities

Volcanos

'.9' ptile of Volcano incidence

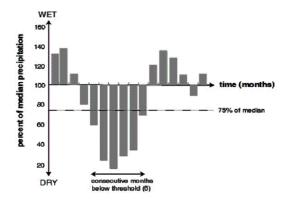


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Introduction and Motivation Data Events Methodology Vulnerabilitie Results

Droughts

Droughts: Classifying a drought.



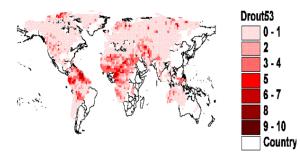
Example of a drought event defined by monthly precipitation being below a threshold of 75% of the long-term median value for at least 3 consecutive months. In this case, the duration of the event was 6 months.

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Introduction and Motivation Data Events Methodology Vulnera Results

Droughts

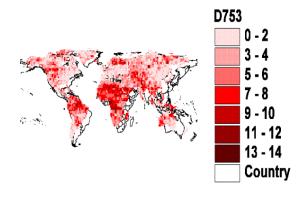
50 pct Weighted Anomaly Standardized Precipitation (WASP)



Introduction and Motivation Data Events Methodology Vulnera Results

Droughts

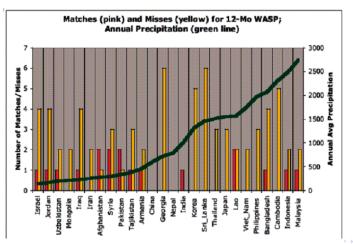
75 pct Weighted Anomaly Standardized Precipitation (WASP)



Introduction and Motivation Data Events Methodology Vulnerat

Droughts

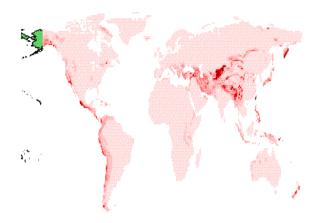
Drought declaration vs. Drought classification



Events Vulnerabilities

Quakes

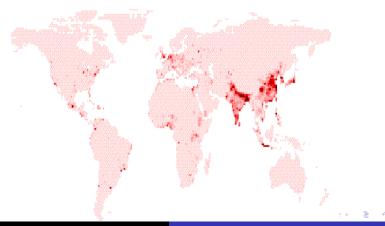
Peak Ground Acceleration



Events Vulnerabilities

Population

Population Density

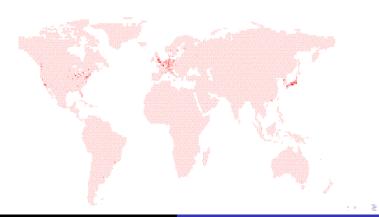


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Events Vulnerabilities

Income





Events Vulnerabilities

Select Bivariate Plots

PGA vs. Floods

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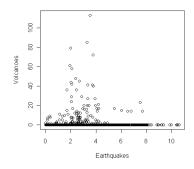
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Events Vulnerabilities

Select Bivariate Plots

PGA vs. Volcanoes



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Multivariate Extreme Value Thresholding

We proceed as follows:

- Select a thresholding level
- Fit an extreme-valued parametric model to the data's tail
- Measure distance between the parametric model and an empirical distribution function

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Approaches to Thresholding

Parametric Model

Asymmetric Logistic Distribution (Tawn 1990):

$$F_{\Theta}(x_1,\ldots,x_d) = \exp\left[-\sum_{b\in B}\left[\sum_{j\in b}\left(\frac{\theta_{j,b}}{y_j}\right)^{1/\alpha_b}\right]^{\alpha_b}
ight]$$

• $j \in \{1, \ldots, d\}$, and y_i is the transformed data

• $B = \text{PowerSet}\{1, \dots, d\} \setminus \emptyset$. Hence, $|B| = 2^d - 1$

• Say, $b = \{2, 4, 7\}$, then the inner sum is over j = 2, 4, 7

• $\alpha_b \in (0, 1] \forall b \in B \setminus B_1$ are dependence parameters

• $\theta_{j,b}$ are asymmetry parameters, with the constraint: $\sum_{b \in B_{(j)}} \theta_{j,b} = 1$ for j = 1, ..., d to force univariate margins to be of the correct form. Here, $B_{(j)} = \{b \in B : j \in b\}$.

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Approaches to Thresholding

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Approaches to Thresholding

Conditional Representation

To derive the pdf, we make use of the positive stable (PS) distribution and its Laplace transform (Stephenson 2009):

- $\int_0^\infty h_1(s) \exp(-st) ds = \exp(-t^\alpha)$
- Take $S_b \sim PS(\alpha_b) \forall b \in B \setminus B_1$, and $S = \{S_b \mid b \in B \setminus B_1\}$. • Then we have for j = 1, ..., d:

$$\mathsf{Pr}(X_j < x_j \mid \mathbf{S} = \mathbf{s}) = \exp \left[-\sum_{b \in B_{(j)}} s_b \left(rac{ heta_{j,b}}{y_j}
ight)^{1/lpha_b}
ight]$$

while X_1, \ldots, X_d are conditionally independent given S = s

• Thus, each marginal asymmetric logistic pdf can be given by:

$$f_j(x_j|s) = \sigma_j^{-1} y_j^{-x_{j_j}} \left[\sum_{b \in B_{(j)}} (z_{j,b}/\alpha_b) \right] \exp\left(-\sum_{b \in B_{(j)}} z_{j,b}\right)$$

where $z_{i,b} = s_b (\theta_{i,b}/v_i)^{1/\alpha_b}$

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Approaches to Thresholding

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MEV Thresholding

Approaches to Thresholding

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Parameter Estimation

- We begin by estimating the marginal parameters

 (μ_j, σ_j, and ξ_j) from univariate data and keep them fixed throughout.
- Simplifying assumptions: we consider high-dimensional (5 and more) asymmetry parameters to be trivial; also, we assume a non-informative prior.
- To obtain estimates for α and θ, we use Metropolis-Hastings within Gibbs to calculate conditional posterior means.

Approaches to Thresholding

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Approaches to Thresholding

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Approaches to Thresholding

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Thresholding

To select the best threshold, we minimize distances between our parametric fit $F_{\hat{\theta}}$ and the empirical distribution function \hat{F}_n – which is given by:



Approaches to Thresholding

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Approaches to Thresholding

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Approaches to Thresholding

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Pickands Type

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Pickands suggesting minimizing KS distance

$$d_k = sup_{f q} |\hat{F}_n({f t}) - \hat{F}_ heta({f t})|$$

ith $k=1,2,...[n/4]$

Joe Type

Joe suggests computing measure of association and setting cutoff to maximize tail dependence

$$max_k \tau_{1-k/n} = max \tau(\mathbf{t}|\mathbf{t} > \mathbf{C}_k)$$
$$= max_k 4E[C_{\theta}(\mathbf{t}|\mathbf{t} > \mathbf{C}_k)] - 1$$

[Joe 1992]

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Approaches to Thresholding

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Generalization of Joe Type

Maximum likelihood over minimum distance:

$$egin{aligned} & {\it max}_{ heta} \; {\it min}_k \; d_{ heta}({f q}, {f C}_{k, heta}) \ & = {\it max}_{ heta} \; {\it min}_k \; {\it E}[{\it ln}(rac{dG_{ heta}({f q})}{dG_{ heta}({f C}_k)})] \end{aligned}$$

Approaches to Thresholding

Kendall's Tau on tails

$ au_{1-k/n}$	$ au_{.9}$	$ au_{.95}$	$ au_{.99}$
Pop-Pga	.072	.186	.472
GNP-Flood	.113	.270	.326
GNP-Drought	.208	.290	.168

70-percentile



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75-percentile



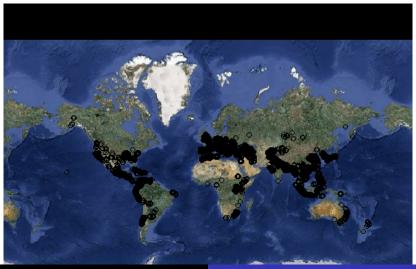
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80-percentile



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85-percentile



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90-percentile



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95-percentile



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99-percentile



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- We fit a flexible model to high-dimensional data.
- This framework allows for the identification of multivariate extremes via either
 - \mathcal{L}^1 or Pickands distance
 - Kullback-Liebler or Expected Entropic Distance.

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- The method (on data ending in 2003) identified several, post hoc, locations → Haiti.
- Compare thresholded 'hotspots' with disaster record from 2003-2010.

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