A Friendly Amendment of the Theil Index

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Theil's Index, a version of Shannon's Entropy, is introduced in econometrics as a measure for inequality. It is improperly specified for statistical use. We explore an adjustment of the Theil index by considering...

- Shannon's original Version
- Theil's (Mis)-Specification via Shannon
- A Friendly Amendment...
- U Mich Health and Retirement Survey (HRS) Data
- A Test for Equi-Inequality?

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Outline

Theil's Version

A version of Theil's index is

$$T = n^{-1} \sum_{i=1}^{n} r_i \, log \, r_i \tag{1}$$

where r_i is the income share for a particular observation (person).

$$r_i = \frac{x_i}{\overline{x}} \tag{2}$$

Theil's Version

Theil's construction suggests

$$H = -\sum_{i=1}^{n} \frac{x_i}{n\bar{x}} \log_b \frac{x_i}{n\bar{x}} \tag{3}$$

With H Shannon's Entropy (more below).

Theil's Decomposition

Thiel's index, on a population of n total individuals, is commonly defined as:

$$T = \sum_{j=1}^{m} p_j \; R_j \; log_b \; R_j + \sum_{j=1}^{m} p_j \; R_j \; T_j \tag{4}$$

with m disjoint groups, $g_1, ..., g_m$, each with n_j members - $n = \sum_j n_j$., and

$$T_j = n_j^{-1} \sum_{i \in g_j} r_{ij} \; log_b \; r_{ij}, \tag{5}$$

Decomposition

Consider this rewrite of (4),

$$T = \sum_G \frac{n_g}{n}\frac{\overline{X}_g}{\overline{\overline{X}}} log_b \frac{\overline{X}_g}{\overline{\overline{X}}} + \sum_G \frac{\overline{X}_g}{\overline{\overline{X}}}\frac{1}{n_g}\sum_g \frac{X_{ig}}{\overline{\overline{X}}_g} log_b \frac{X_{ig}}{\overline{\overline{X}}_g} \qquad (6)$$

Decomposition

The first term on the right hand side

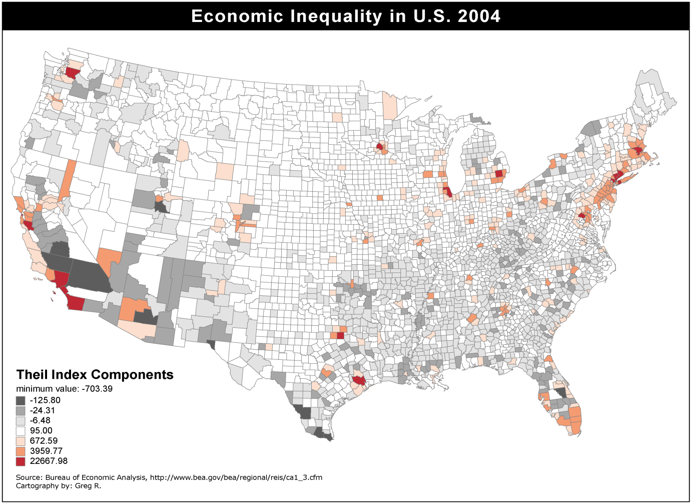
$$Across = \sum_{G} \frac{n_j}{n} \frac{\overline{X}_g}{\overline{X}} \, log_b \frac{\overline{X}_g}{\overline{X}} \tag{7}$$

is the measure of the *across* or *between* group inequality; the second term

$$Within = \sum_{G} \frac{\overline{X}_g}{\overline{X}} \frac{1}{n_g} \sum_{g} \frac{X_{ig}}{\overline{X}_g} log_b \frac{X_{ig}}{\overline{X}_g} \tag{8}$$

the *within* group inequality.

Illustration



Shannon's Entropy

Shannon represents an 'Information Source' - a random process on a discrete space - as a Markov process.
He supposes a "measure" H should have these qualities:

1. H should be continuous in event probability, p_i
2. For $p_i = p$, $\forall\ i$, H should monotonically increase
3. "If a choice be broken down, the original [H] should be the weighted sum"

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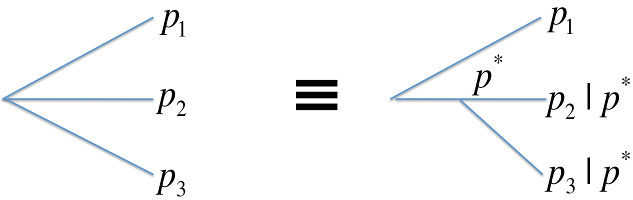
"If a choice be broken down"

Consistency over conditioning...

$$H(p_1, p_2, p_3) \equiv H(p_1, p^*) + p^* H(p_2|p^*, p_3|p^*)$$



Shannon's Theorem

The only H satisfying the three axioms is of the form (1949)

$$H = -K \sum_{i=1}^{n} p_i \; log \; p_i$$

Other 'Entropies'

- Gibbs Entropy: $S = -k_B \sum p_i \, log \, p_i$; (1872, 1878)
- von Neumann Entropy: $S - k_B Tr[\rho \, log_e \, \rho]$

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Entropy's Career

Physics	Electrical Engineering → Computing → Computer Science
Statistics	Frechet → Ash → Kullback → Rissanen
Humanities	Theil: Econometrics

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Outline

Theil's Version

Since

$$\frac{log_{b_0}\ t}{log_{b_1}\ t} = K,\ \forall t \tag{9}$$

say, K can serve as a conversion between information units b_0 and b_1. **This feature is elided from Theil's construction.**

Misspecification

Examine the expectation of the *within* term:

$$E[Within] = E[\sum_G \frac{1}{\overline{X}n_g} \sum_g E[X_{ig} log_b \frac{X_{ig}}{\overline{X}_g} | G = g]] \tag{10}$$

$$= E[\sum_G \frac{1}{\overline{X}n_g} E[\sum_g X_{ig} log_b \frac{X_{ig}}{\overline{X}_g}]] \tag{11}$$

$$\geq E[\sum_G \frac{1}{\overline{X}n_g} E[n_g \overline{X}_g log_b \frac{n_g \overline{X}_g}{n_g \overline{X}_g}]] \tag{12}$$

$$\geq E[\sum_G \frac{1}{\overline{X}n_g} E[0]] = 0 \tag{13}$$

Misspecification

An inspection of the across term,

$$E[Across] = E[\sum_G \frac{n_g}{n} \frac{\overline{X}_g}{\overline{X}} \, log_b \frac{\overline{X}_g}{\overline{X}}] = E[\sum_G \alpha_g \, \beta_g^b] \tag{14}$$

Misspecification

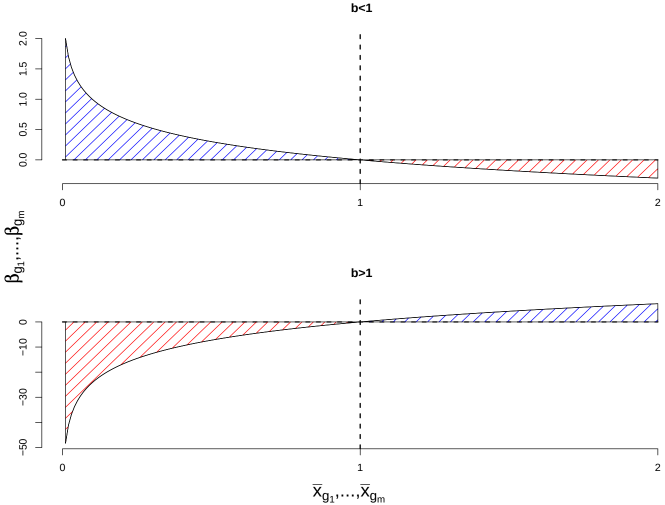
An inspection of the across term,
Note that $\sum_G \alpha_g = 1$, and $\alpha_g \geq 0$, by assumption, $\forall g$. Then,

$$E[Across] = E[\sum_G \alpha_g \, \beta_g^b] \quad (15)$$

$$\leq E[\sum_G \alpha_g \, \sum_G \beta_g^b] = E[1 \cdot \sum_G \beta_g^b] \quad (16)$$

$$= E[\sum_G \beta_g^b] \quad (17)$$

Illustration



Misspecification

In some situations

$$E[\sum_G \beta_g^b] = E[log_b(\frac{\prod_G \overline{X}_g}{\overline{X}^b})] \leq 0 \qquad (18)$$

The bound for the expected value of the across partition of Theil's index can be made positive or negative, arbitrarily, depending upon choice of log base *b*.
Theil himself is indeterminate on the importance of *b*

Misspecification

- Choosing b is like choosing 'natural' ratio of group to overall inequality
- The effect is inflated in data where few, small groups (n_g) have high, or low, income relative to population size.
- The effect in b is non-linear b
- While contribution, n_g, is only linear.

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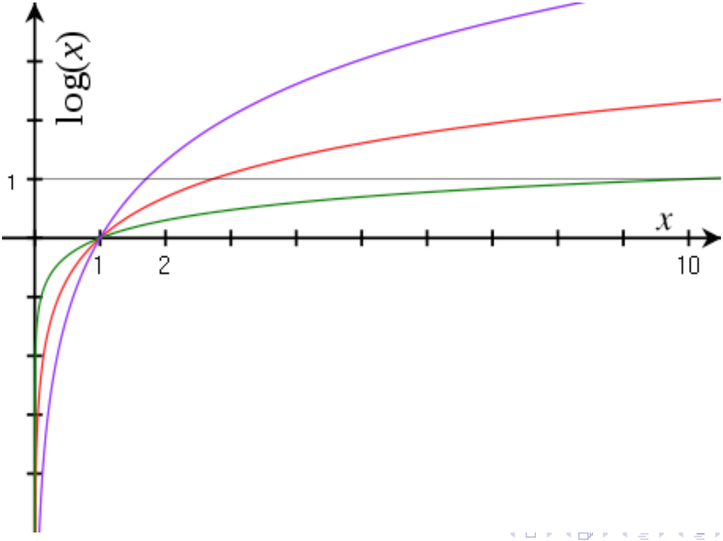

Comments

- The probability of a particular event/realization is replaced with the income share for a particular individual.
- We were measuring prob mass of events, now we are measuring the ‘size’ of individual
- In this sense: The individual is the event, and the income share is the prob mass

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Technical Comments

- The lack of a true event space (σ-algebra) yields a degenerate probability model...
- Though heuristic is consistent: *n*-dimensional simplex / Liouville family of distributions.
- But this belies a "deeper confusion" about Entropy and Probability [Jaynes (1965), *American Journal of Physics*].

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Illustration

$$\Omega = \left\{ E_1 \quad E_j \quad E_n \right\}$$

$\{$

X_1

X_j

X_n

$\} = ?$

Technical Comment: The Amnesia of (Probability) Measure

- The problems don't arise on Shannon's specification via the unit simplex
- Theil Index is on simplex of arbitrary sum
- Theil Index is an *ex parte* function on probability measures

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Approaches

- **Adjust** b
- **Rank Transform** Estimate empirical CDF across and within groups and substitute for raw income shares

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- **Rank Transform** Estimate empirical CDF across and within groups and substitute for raw income shares

Adjust b

In many settings — Rocke and Durbin Log-Linear hybridization; Box-Cox, Tukey families of transforms.. — b is a parameter to be estimated. $b = inf_g(R_g)$
Set

$$\hat{b} = min(r_g) = min_g(\frac{\overline{x}_g}{\overline{x}})$$

the minimum group income share.

Fix b

Set

$$b \leq \min_G(\overline{X}_g/\overline{X}) \implies E(\sum_G \beta_g^b) \geq 0 \quad (19)$$

or

$$b \geq \max_G(\overline{X}_g/\overline{X}) \implies E(\sum_G \beta_g^b) \leq 0 \quad (20)$$

Rank Transform

Essentially this is to revert Theil to Shannon...where $F_g = \mathbb{P}(group\ g)$ and $F_{X_g} = \mathbb{P}(X_g < x_g)$.
Set

$$Across \equiv \sum_g \hat{F}_g \cdot log_b(\hat{F}_g)$$

and

$$Within \equiv \sum_g \hat{F}_g \sum_{X_g} \hat{F}_{X_g} \cdot log(\hat{F}_{X_g})$$

Use $0 \leq \hat{F}_g, \hat{F}_{X_g} \leq 1$ or $\hat{F}_g, \hat{F}_{X_g} \in \{1, ..., m\}, \{1, ..., n_g\}$

Outline

A T statistic

Consider a hypothesis testing setup for the inequality of the income distribution in a population

- For any $b > 0$ the expected value of Theil's index T is zero.
- Note that this is the value of T for a uniform distribution of incomes

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A T statistic

In this setup we can treat the observed value of T, as a test statistic for the hypothesis of inequality:

$$\begin{aligned} H_0 &: \mathcal{T} = 0 \\ &vs. \\ H_a &: \mathcal{T} > 0 \end{aligned} \tag{21}$$

Dist of T

Martinez-Camblor suggests that the observed value, or estimator, T is asymptotically normal about T. Using this finding yields

$$p - value = \mathbb{P}_{H_0:\mathcal{T}=0,b}\left(Z > \frac{T}{s.e.(T)}\right) \qquad (22)$$

with Z a standard normal Gaussian random variable and $s.e.(T)$ the standard error of the estimator, estimated via bootstrapping.

Testing via T

We suggest simultaneous testing of across and within group inequality:

$$\begin{aligned} H_0 : \mathcal{T}_a = 0 = \mathcal{T}_w \\ vs. \\ H_a : \mathcal{T}_a > 0; \mathcal{T}_w \neq 0 \end{aligned} \tag{23}$$

Given an appropriate b, this is

$$\begin{aligned} H_0 : \mathcal{T}_a = 0 = \mathcal{T}_w \\ vs. \\ H_a : \mathcal{T}_a > 0; \mathcal{T}_w > 0 \end{aligned} \tag{24}$$

Outline

Health and Retirement Survey

We offer an illustration using the University of Michigan's Health and Retirement Survey (HRS) data.
The data are a battery of responses from a well known longitudinal study (see Emelech 2006) in particular investigates intra-group inequality for white, black and Latino Americans.

Health and Retirement Survey

Using data from the 2000, 2002, 2004, 2006 and 2008 waves of the survey we calculate the Theil indexes on wealth and income household totals, grouping on ethnicity.
We chose to ignore the Hispanic/Latino classification for this illustration. This grouping yielded 19580, 18167, 20134, 18469 and 724 observations by increasing year.

Health and Retirement Survey

- We used responses to total net worth and total income.
- We case-wise deleted values from the data that were less than zero; by assumption here and elsewhere Theil's index is calculated solely on positive quantities.
- Additionally, after removing cases with negative values in either wealth or income, we shifted all responses by one.
- This may *lessen* the appearance of inequality in the data

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Illustration

Illustration of Theil's index calculated on wealth - left hand column - and income - right hand column - using the University of Michigan's Health and Retirement Survey (HRS) data: 2000, 2002, 2004, 2006, and 2008.

Illustration

- The upper row is the across term, the lower row is the within term.
- Both terms are fixed by log base $b = \min(\overline{x}_g/\overline{x})$ the ratio of the poorer (black) group sample mean to the overall mean.
- The small number of observations in 2008 (n=724) inflate the (estimate of) standard error - via ordinary bootstrap.
- Across group inequality appears to be stable or decreasing from 2000-2008; within group inequality appears to be increasing from 2000-2006.
- Confidence bars are at 95 percent significance.

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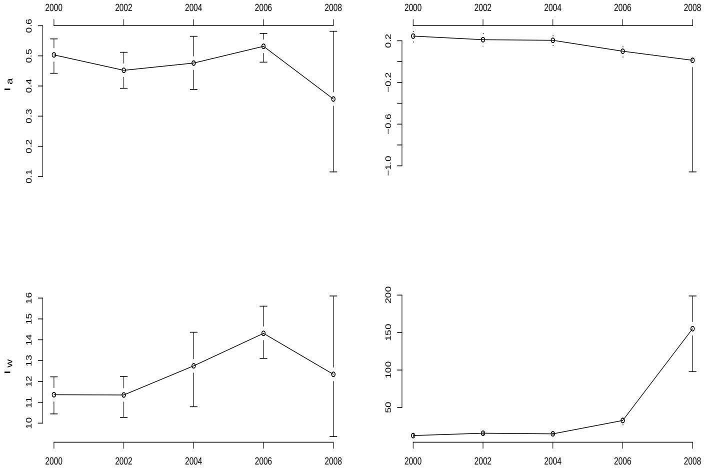

Illustration

- The Figure plots the across and within observed Theil indexes, by year.
- The small number of data in 2008 inflates the estimate of the standard error - obvious from the illustration by the wider confidence bars around the estimate.
- In general the observed estimate of across group inequality from 2000-2008 appears to be decreasing for both wealth and income.
- However - in the lower panel of the figure - the within group inequality sum appears to (mainly) increase from 2000-2008 (the 2008 estimate may be unreliable).

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- Across group wealth and income inequality appears to be decreasing from 2000-2008, yet across group wealth inequality seems to be noticeably greater than income inequality.
- Over the same period within group inequality, via both wealth and income, appears to be increasing. This agrees with contemporary research (see Darity 2000 and Emelech 2006).
- While differences across groups appear to be lessening - within racial group stratification is worsening.

Furthermore, it is important to notice that within group income disparity - the lower right panel in the figure - is larger in magnitude and change.

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Z statistics

Year	Across	Within
2000	12.2	19.8
2002	7	22.1
2004	10	30.8
2006	4.0	13.9
2008	.0034	5.45

Z Statistics

Test statistics, Z_o, for T_a, Theil's index for calculated for income on the HRS data from 2000-2008. Compare each with a quantile from the standard normal distribution - for either of $H_0 : T_a, T_w = 0$, $Z_o \geq 1.64$ implies significance at the .05 level.

Z Statistics

Year	Across	Within
2000	25	23.6
2002	15	18.6
2004	9.4	13.7
2006	20.4	22.16
2008	2.97	7.38

Z Statistics

Test statistics Z_o, for T_w, Theil's index calculated for wealth on the HRS data from 2000-2008. Compare each with a quantile from the standard normal distribution - for either of $H_0 : T_a, T_w = 0$, $Z_o \geq 1.64$ implies significance at the .05 level.

Outline

Equi-Inequalities

Several hypothesis of Equi-Inequality to consider:

I Each group is equally unequal

II Groups are equal

III Each group contributes equally to inequality measure

IV Inequality Across Groups is equivalent to Inequality Within Groups

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I. Each group is equally unequal
II. Groups are equal
III. Each group contributes equally to inequality measure
IV. Inequality Across Groups is equivalent to Inequality Within Groups

Hypothesis I, say

Take hypothesis I, for example, i.e. $T_g = T \; \forall g$

$$\frac{Across}{Within} \to T^{-1}\left(\frac{\sum_g n_g \overline{X}_g log_b(\overline{X}_g)}{\sum_g n_g \overline{X}_g} - log_b(\overline{X})\right)$$

Hypothesis I, say

Since T, $log_b(\overline{X})$ and are ancillary under I, the test statistic of interest is

$$\Phi_I \equiv \frac{\sum_g n_g \overline{X}_g log_b(\overline{X}_g)}{\sum_g n_g \overline{X}_g}$$

Hypothesis II, say

This is equivalent to the ordinary ANOVA or Bartlett's test for means or variances.

Hypothesis III, say

This is a generalization of I; the test statistic of interest is

$$\Phi_{III} = \sum_{g} \frac{n_g}{n} \frac{\overline{X}_g}{\overline{\overline{X}}} log_b(\frac{\overline{X}_g}{\overline{\overline{X}}})$$

Under III

$$\Phi_{III} \rightarrow m \cdot C$$

Hypothesis IV

This is the most general hypothesis; the test statistic is the ratio of across to within

$$\Phi_{IV} = \frac{\sum_G \frac{n_g}{n} \frac{\overline{X}_g}{\overline{\overline{X}}} \; log_b \frac{\overline{X}_g}{\overline{\overline{X}}}}{\sum_G \frac{\overline{X}_g}{\overline{\overline{X}}} \frac{1}{n_g} \sum_g \frac{X_{ig}}{\overline{X}_g} log_b \frac{X_{ig}}{\overline{X}_g}}$$

If b is well chosen, this a ratio of two positive quantities that is 1 at the null hypothesis. Similar to an F statistic.

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Illustration