CAARMS Talk

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1. Part I - Introduction and History.
2. Part II - Welfare Economics, Sustainable Criteria.
3. Part III - Statistics.

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Part I

Introduction and History

What is Sustainability?

- Various definitions from various authors and settings.
- Generally a **process characterization.**
- To reform Emery's prurient simile: Sustainability is hard to define but obvious when it is absent.

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History of ‘Sustainability’

Thomas Malthus’ *An Essay on the Principle of Population* (1798-1826 six editions)

“Of Increasing Wealth, as it affects the Condition of the Poor”

History of 'Sustainability'

Frank Ramsey's *A Mathematical Theory of Saving* (1928)

"[interest on wealth] is ethically indefensible...polite expression for rapacity..."

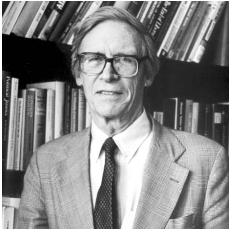
History of 'Sustainability'

Harold Hotelling's *The Economics of Exhaustible Resources* (1931)

"Problems of exhaustible assets are peculiarly liable to become entangled with the infinite."

History of ’Sustainability’

John Rawls’ *Theory of Justice* (1972)



“Just society is one so organized as to promote the greatest...well being of least well off.”

Early Themes

- Implicit Concern for [intergenerational] social welfare
- Explicit worry about consumption
- Beginnings of mathematical formalism, but...
- ...mostly 'theological' and narrative.
- Trade-off between wealth [consumption] and sustainability.

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Working Definition

A **Sustainable** process: One that can be maintained at a certain level indefinitely.
Sustainable Development: Development of economic systems that can last indefinitely.
Bruntland Commission (1983): *"Meets the needs of the present without compromising the ability of future generations to meet their own needs."*

An Aside: The ESI

Environmental Sustainability Index 2002

Part II

A Sustainable Criteria

Mathematical Formalism: Welfare Economics

A simple setup, let:

- c_t the consumption of some resource at time t
- s_0 initial stock of resource.
- $u(c_t)$ an increasing, strictly concave function ($\dot{u} > 0$, $\ddot{u} < 0$).

Mathematical Formalism: Welfare Economics

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Welfare Economics

The goal under Hotelling's version of **discounted utilitarianism** (1931) is to:

$$max\ W(u) = max \int_0^\infty u(c_t)e^{\delta t}dt$$

s.t.

$$\int_0^\infty c_t \leq s_0$$

Welfare Economics

Write:

$$s_t = s_0 - \int_0^t c_f df$$

$$s_t \geq 0 \ \forall \ t$$

Welfare Economics

Yielding...

$$max\ W(u) = max \int_0^{\infty} u(c_t)e^{-\delta t}$$

s.t.

$$s_t \geq 0$$

and

$$\dot{s}_t = -c_t$$

Welfare Economics

Hamiltonian approach: Add to maximand the RHS of the differential equation constraint times multiplier.

$$H = u(c_t)e^{-\delta t} - \lambda_t e^{-\delta t} c_t$$

with λ_t the **shadow price** at time t and $\lambda_t e^{-\delta t}$ the present value shadow price.

Welfare Economics

Maximizing H w.r.t. c_t yields

$$\dot{u}(c_t) = \lambda_t \; \forall t$$

and

$$\frac{d(\lambda_t e^{-\delta t})}{dt} = -\frac{\partial H}{\partial t}$$

Welfare Economics

These imply

- The increase in utility from consumption should equal the shadow price of the resource.
- $\frac{d\lambda}{dt} - \delta\lambda = 0 \rightarrow \lambda_t = \lambda_0 e^{\delta t}$.

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Hotelling Rule

- Consumption is regulated by change in (shadow) price.
- Difference in welfare functions is governed by state variable s_0 and discount rate parameter δ.

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Variations on measuring utility

Malthus:

$$W(u;\delta)^{Malthus} := max_u \left\{ min_{c_t} \int_0^\infty u(c_t) e^{-\delta t} dt \right\}$$

Variations on measuring utility

Ramsey:

$$W(u;\delta,B)^{Ramsey} := min \int_0^\infty [B - u(c_t)]dt$$

where $B = sup_t u(c_t)$.

Variations on measuring utility

Rawls:

$$W(u;\delta)^{Rawls} := max_u \left\{min_t u(c_t)e^{-\delta t}\right\}$$

Objectives under previous criteria

- The constraint $\int_0^\infty c_t dt \leq s_0$ applies to all of the above.
- The shadow prices of the resources, λ_0, oppose consumption - with additional parameters (B, δ for example).
- Thinking statistically, the criteria are **estimating equations** for parameters.

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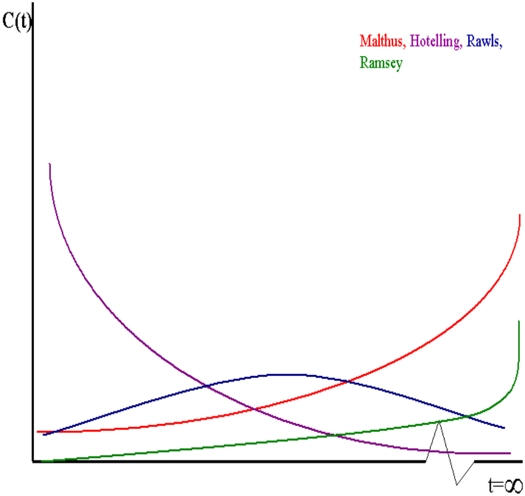
Objectives under previous criteria

The criteria yield this inequality:

$$\lambda_0^{Rawls} > \lambda_0^{Ramsey} > \lambda_0^{Malthus} > \lambda_0^{Hotelling}$$

So the relevant effect - for sustainability - is to raise the shadow price of the exhaustible resource.
And the relevant effect - for statistics - is to define consumption (and discount rate) as solution to estimating equations

Comparison of criteria



Insensitivity to Present or Future

Notice...

- ...the Malthus and Ramsey welfare functions ‘weigh’ the infinite future greater than the finite present.
- ...the Hotelling and Rawlsian criteria ‘weigh’ the finite present greater than the infinite future.

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Part III

Statistical Methods for Sustainablility

Sustainability Characterized

Chichilnisky (1997) introduces an axiomatization for intergenerational equity as a criteria for sustainability:

Sustainability Characterized

Letting $\mathbf{u} = \{u_t\}_{t=1}^{\infty}$ be a bounded, real valued utility stream - on $(\Omega, \mathcal{F}_t, \mathbb{P} = \mathbb{P}^* + \mathbb{P}_\infty)$.

- **Insensitivity to the future**: $\mathbb{P}_\infty(W(\mathbf{u})) = 0$ if $\mathbb{P}_\infty$ is a *purely finitely additive* measure.
- **Insensitivity to the present**: $E_{\mathbb{P}^*}[W(\mathbf{u})] = 0$ for all countably additive measures $\mathbb{P}^*$

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Chichilnisky's axiomatization

Chichilnisky's axiomatization [1996,1997]:

$$W(\mathbf{u}) = \alpha \int_{R^+} u(c_t) d\mathbb{P}^*(t) + (1-\alpha)\mathbb{P}_\infty(\mathbf{u})$$

where $\mathbb{P}_\infty$ is measure zero on finite sets. This characterization ensures equity to present and unforseen generations.

Chichilnisky's axiomatization

The combination of measures which are singular w.r.t each other disallows ordinary optimization procedures.

$$\mathbb{P}^* \perp \mathbb{P}_\infty$$

Chichilnisky's axiomatization

However:
Both $d\mathbb{P}^*$ and $d\mathbb{P}_\infty$ are absolutely continuous with respect to $d\mathbb{P}$

Statistical Estimation

The goal here is to introduce statistical estimators for a sustainable development path - or utility stream - via a representation of Kullback-Leibler divergence.

Probability Measure Based Distance

Call:

$$D(d\mathbb{P}^k, d\mathbb{P}) = E_{d\mathbb{P}}[log(\frac{d\mathbb{P}^k}{d\mathbb{P}})]$$

Probability Measure Based Distance

Let $d\mathbb{P}^k$ be the probability density induced by the filtration $\mathcal{F}$ at the $k-th$ cutoff of the utility stream.
Then

$$KL(d\mathbb{P}^k, d\mathbb{P}) = \sum^{k} d\mathbb{P} log(\frac{d\mathbb{P}_k}{d\mathbb{P}})$$

Probability Measure Based Distance

But $d\mathbb{P} = d\mathbb{P}^{*} + d\mathbb{P}_{\infty}$. So

$$KL(d\mathbb{P}^{k}, d\mathbb{P}) =$$

$$= \sum^{k} d\mathbb{P}^{*} log(\frac{d\mathbb{P}^{k}}{d\mathbb{P}^{*} + d\mathbb{P}_{\infty}}) + \sum^{k} d\mathbb{P}_{\infty} log(\frac{d\mathbb{P}^{k}}{d\mathbb{P}^{*} + d\mathbb{P}_{\infty}})$$

Probability Measure Based Distance

$$= \sum^{k} (d\mathbb{P}^* + d\mathbb{P}_\infty) log(d\mathbb{P}_k) - \sum^{k} (d\mathbb{P}^* + d\mathbb{P}_\infty) log(d\mathbb{P}^* + d\mathbb{P}_\infty)$$

The second term is just the entropy of the full measure. Looking at the first term...

Probability Measure Based Distance

...and taking the conditional expectation

$$= E(\sum^{k}(d\mathbb{P}^{*} + d\mathbb{P}_{\infty})log(d\mathbb{P}_{k})|\mathcal{F}_{k})$$

$$= E(\sum^{k} d\mathbb{P}^{*} log(d\mathbb{P}_{k})|\mathcal{F}_{k}) + E(\sum^{k} d\mathbb{P}_{\infty} log(d\mathbb{P}_{k})|\mathcal{F}_{k})$$

Probability Measure Based Distance

yields

$$= \sum^{k} log(d\mathbb{P}_k)E(d\mathbb{P}^*) + \sum^{k} log(d\mathbb{P}_k)E(d\mathbb{P}_\infty)$$

This is $:= data \cdot parameters + data \cdot parameters$, as well as a convex sum of singular measures meeting Chichilnisky's criteria.

Probability Measure Based Distance

Minimizing

$$= \sum^{k} log(d\mathbb{P}_k)E(d\mathbb{P}^*) + \sum^{k} log(d\mathbb{P}_k)E(d\mathbb{P}_\infty)$$

with respect to the parameters of the measures (which can include mixing parameter α) yields estimating equations which yield inference on utility/developments (i.e. consumption) paths

Next steps

- Derive examples for singular measure pairs
- Investigate distribution of D, estimating equations, possible CUSUM test.
- Thank CAARMS for the last speaking spot.

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